Self-support of Japanese astronomy in Edo Period

- *Seki Takakazu* (關孝和) and *Shibukawa Harumi* (澁川春海) -

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Abstract

In pre-Edo period of Japan, five systems of Chinese calendar were officially used, namely the *Yuanjia-li*, the *Yifeng-li*, the *Dayan-li*, the *Wuji-li* and the *Xuanming-li*. Among them, the last Chinese calendrical system used in pre-Edo period of Japan was the *Xuanming-li*, which was used from AD 862 to 1684 in Japan.

By the beginning of the Edo period (AD 1603 − 1867), some Japanese scholars noticed the inaccuracy of the old *Xuanming-li*, and tried to study the more accurate *Shoushi-li*, an excellent Chinese traditional calendrical system which was made by *Guo Shoujing* etc. at the time of the *Yuan* dynasty of China, and was used in China from AD 1281 (but has never been used officially in Japan). Seki Takakazu (ca.AD 1640 − 1708), a celebrated mathematician, and Shibukawa Harumi (AD 1639 − 1715), an able practical astronomer, also studied the *Shoushi-li*.

Shibukawa Harumi, as a practical astronomer, found that even the *Shoushi-li* was not accurate enough, and tried to use new data including fragmental information of Western astronomy. In AD 1683, *Shibukawa* proposed his own new calendrical system, which was named *Jōkyō-reki* in the next year, and was officially used in Japan from AD 1685. It was the first theory of calendrical system produced in Japan.

Seki Takakazu and *Shibukawa Harumi* were contemporaneous, and both of them made great contributions to the development of Japanese mathematics and/or astronomy. I would like to discuss the difference between *Seki Takakazu* and *Shibukawa Harumi* in the study of astronomy.

1. Introduction

In pre-Edo period of Japan, five systems of Chinese calendar were officially used, namely the *Yuanjia-li* (元嘉暦, *Genka-reki* in Japanese), the *Yifeng-li* (儀鳳暦, *Gihō-reki* in Japanese ; which is the same as the *Linde-li* （麟徳暦, *Rintoku-reki* in Japanese）), the *Dayan-li* (大衍暦, *Taien-reki* or *Daien-reki* in Japanese), the *Wuji-li* (五紀暦, *Goki-reki* in Japanese) and the *Xuanming-li* (宣明暦, *Senmyō-reki* in Japanese). Among them, the last Chinese calendrical system used in pre-Edo period of Japan was the *Xuanming-li* (Chinese *li* or Japanese reki stands for "calendar" （or ephemeris）), which was used from AD 862 to 1684 in Japan.

By the beginning of the Edo (江戸) period (AD 1603 – 1867), some Japanese scholars noticed the inaccuracy of the old *Xuanming-li*, and tried to study the more accurate *Shoushi-li* (授時暦, Juji-reki in Japanese), an excellent Chinese traditional calendrical system which was made by *GUO Shoujing* (郭守敬) etc. at the time of the *Yuan* (元) dynasty of China, and was used in China from AD 1281 (but has never been used officially in Japan). *Seki Takakazu* (關孝和, ca. AD 1640 – 1708) and *Shibukawa Harumi* (澁川春海, AD 1639 – 1715) also studied the *Shoushi-li*.

In AD 1683, *Shibukawa Harumi* proposed his own new calendrical system, which was named *Jōkyō-reki* (貞享暦) in the next year, and was officially used in Japan from AD 1685. It was the first theory of calendrical system produced in Japan.

Seki Takakazu and *Shibukawa Harumi* were contemporaneous, and both of them made great contributions to the development of Japanese mathematics and/or astronomy. I would like to discuss the difference between *Seki Takakazu* and *Shibukawa Harumi* in the study of astronomy.

2. The *Xuanming-li* and the *Shoushi-li*

2.1 The *Xuanming-li* (*Senmyō-reki* in Japanese)

The *Xuanming-li* (宣明暦) is a calendrical system compiled by a Chinese astronomer *Xu Ang* (徐昂) at the time of *Tang* (唐) Dynasty of China, and was used from AD 822 to 892 in China. It was also used from the end of the *Silla* (新羅) Dynasty to the *Koryo* (高麗) Dynasty in Korea. It was also used in Japan from AD 862 to 1684.

2.2 The *Shoushi-li* (*Juji-reki* in Japanese)

The *Shoushi-li* (授時暦) is a calendrical system compiled by a Chinese astronomer *Guo shoujing* (郭守敬) (AD 1231 – 1316) and his colleagues at the time of *Yuan* (元) Dynasty of China, and was used from AD 1281 to 1367 (and, after that, its emendation *Datong-li* (大統暦) was continually used) in China. The *Shoushi-li* (or its emendations or revisions) was also used from the end of the *Koryo* (高麗) Dynasty to the middle of the *Choson* (朝鮮) Dynasty in Korea. However, it has never been used officially in Japan.

2.3 Accuracy of the *Xuanming-li* and the *Shoushi-li*

In solar and lunar theories, the most fundamental constants are the length of a tropical year (回帰年, from winter solstice to winter solstice) and of a synodic month (朔望月, from new moon to new moon). As the solar motion and the lunar motions are not uniform, their inequalities (which correspond to the modern “equation of centre”) are applied in order to calculate the time of new moon or full moon. In the case of the moon, an anomalistic month (近点月, from [lunar passage of] perigee to perigee) is the cycle of this correction. In the case of the sun, the perigee is considered to be at the point of winter solstice (first point of Capricorn) in Chinese traditional calendars such as the *Xuanming-li* and the *Shoushi-li*. So, an anomalistic year (近点年, from [solar passage of] perigee to perigee) in these calendars was the same as a tropical year. For the prediction of eclipses, the length of a nodical month (交点月, from [lunar passage of] a node to the same node) should be known. A lunar node is an intersection of the lunar orbit and the solar orbit (ecliptic). Due to the precession of the equinoxes, the length of a sidereal year (恒星年, period of solar revolution with reference to the fixed stars) is slightly longer than that of a tropical year.

The modern exact value of some astronomical constants is as follows.

Lunar constants:

Synodic month ≈ 29.53059 days,

Anomalistic month ≈ 27.55455 days,

Nodical month ≈ 27.21222 days.

Solar constants:

Tropical year ≈ 365.2422 days,

Anomalistic year ≈ 365.2596 days,

Sidereal year ≈ 365.2564 days.

Let us first see the accuracy of the linar theory.

In the *Xuanming-li*, one day is divided into 8400 parts, and some of the constants are expressed by fraction whose denominator is 8400.

The lunar constants in the *Xuanming-li* and their error in a century are as follows.

$$\text{Synodic month} = \frac{248057}{8400} \text{ days} \approx 29.53060 \text{ day,}$$

(Error: about +0.01 day in a century.).

$$\text{Anomalistic month} = \frac{231458\frac{19}{100}}{8400} \text{ days} \approx 27.55455 \text{ days,}$$

(Almost exact.).

$$\text{Nodical month} = \frac{228582\frac{6512}{10000}}{8400} \text{ days} \approx 27.21222 \text{ days,}$$

(Almost exact.).

In the *Shoushi-li*, one day is divided by the powers of ten, and the expression is practically similar to the decimal fraction system.

The lunar constants in the *Shoushi-li* and their error in a century are as follows.

Synodic month = 29.530593 days, (Almost exact.).

Anomalistic month = 27.5546 days, (Error: about +0.07 day in a century.).

Nodical month = 27.212224 days, (Almost exact.).

From the above data, it is clear that these constants are enough accurate in the *Xuanming-li* and the *Shoushi-li*, and these calendars can be used for centuries.

Now let us see the accuracy of the solar theory.

The solar constants in the *Xuanming-li* and their error in a century are as follows.

$$\text{Tropical year} = \frac{3068055}{8400} \approx 365.2446 \text{ days,}$$

(Error: about +0.24 day in a century.).

$$\text{Sidereal year} = \frac{920446199}{2520000} \approx 365.2564 \text{ days,}$$

(Almost exact.).

We should note that the length of a tropical year in the *Xuanming-li* is slightly longer than the exact value.

The solar constants in the *Shoushi-li* and their error in a century are as follows.

Tropical year = 365.2425 days, (Error cannot be expressed in simple way, but is almost exact.).

Sidereal year = 365.2575 days, (Error: about +0.11 day in a century.).

In the *Shoushi-li*, the length of a tropical year diminishes by 0.0002 day in a century by its *"xiaozhang-fa"* (消長法). This rate is too large, and the length of a tropical year becomes too short if it is used for several centuries. Fortunately, the length of a tropical year in the *Shoushi-li* is slightly longer than the exact value (365.2422 days), and the effect of the *"xiaozhang-fa"* did not produce much error for a few centuries, but the Edo period was already the time when the *"xiaozhang-fa"* had been producing certain error, of which *Shibukawa Harumi* was not aware. *Shibukawa Harumi* succeeded the *"xiaozhang-fa"* in his *Jōkyō-reki*.

Now let us discuss the problem of the solar perigee (which corresponds to the modern perihelion of the Earth in the opposite direction in the space). The longitude of the solar perigee (observed from the Earth) in the year t is as follows.

$279^\circ 30' 12''.30 + 61''.8026\ t + \ldots$, (epoch Gregorian 1800 January 0, noon).

So, the solar perigee can roughly be expressed as follows.

$279^\circ.5 + 1^\circ.7\ T$, $[T \equiv (t - 1800)/100]$.

From this expression, we know that the solar perigee was at the point of winter solstice (first point of Capricorn) in mid 13thcentury, that is the time of the preparation of the *Shoushi-li*. The *Shoushi-li* considered that the solar perigee was fixed to the point of winter solstice. It was right at the time of the *Shoushi-li*. However, in mid 17thcentury when *Shibukawa Harumi* was studying calendar, the true longitude of solar perigee was about $7^\circ.5$ from the point of winter solstice. So, the *Shoushi-li* was quite innacurate in this respect.

In the case of the *Xuanming-li*, the true longitude of the solar perigee was about -7° from the point of winter solstice when it was made in the 9th century. As the *Xuanming-li* also considered that the solar perigee is at the point of winter solstice, it was somewhat inaccurate when it was made. Luckily its inaccuracy diminished for 5 centuries or so, and then started to increase. In this respect, the *Xuanming-li* was quite fortunate. What was more fortunate is that the length of a tropical year in the *Xuanming-li* was too long. If the length of a tropical year in the calendar is too long, the solar perigee in the calendar, which is fixed to the point of calendrical winter solstice, advances among the actual sky, and this inaccuracy produces an effect that the longitude of the solar perigee in the calculation of eclipses etc. practically increases. As the time of the winter solstice according to the *Xuanming-li* was about 2 days later than the true winter solstice at the time of *Shibukawa*, we can consider that the practical longitude of the solar perigee in the *Xuanming-li* was about 2° from the true point of winter solstice at that time. As the correct longitude was about 7°.5 from the point of winter solstice, the amount 2° is much better than the amount 0° in the *Shoushi-li* which was quite accurate otherwise.

So, at the beginning of Edo period, the *Xuanming-li* sometimes could produce better result than the *Shoushi-li* thanks to the inaccuracy of the length of a tropical year in the *Xuanming-li* as far as the prediction of eclipses is concerned. This must be one reason why *Shibukawa* recognized that even the *Shoushi-li* was not enough accurate.

3. *Seki Takakazu* the Mathematician

Seki Takakazu (關孝和, ca.AD 1640 – 1708) was a celebrated mathematician, and wrote several mathematical works. He is the most famous mathematician in the Edo period. He is the founder of a mathematical school “*Seki* school” (關流), and Japanese mathematics highly developed in this school. Besides the mathematical works, he wrote some astronomical works also.

Seki Takakazu understood the theoretical aspect of the *Shoushi-li*’s mathematical astronomy quite well, and his mathematical ability seems to have been superior to that of *Shibukawa*. However, it can be said that *Seki* was a theoretician rather than a practician. As far as astronomy is concerned, *Seki* was almost independent of Western influence, whilst *Shibukawa* accepted certain Western knowledge such as the round earth theory etc. *Seki* did not try to revise the *Shoushi-li*, while *Shibukawa* did.

4. *Shibukawa Harumi* the Astronomer

Shibukawa Harumi (澁川春海, AD 1639 – 1715), who was originally a “go” (碁, a kind of Japanese game) player, was also interested in astronomy, and studied the *Shoushi-li*. Besides the information through the books from China, certain informathion about the *Shoushi-li* was also brought to Japan through a traveller from Korea, as the *Shoushi-li* was well studied in Korea at that time. *Shibukawa* noticed the innacuracy of the *Xuanming-li*, and tried to substitute the *Shoushi-li* for the *Xuanming-li*. However, it was found that even the *Shoushi-li* was not enough accurate.

In AD 1683, *Shibukawa Harumi* proposed a new calendrical system, which was named *Jōkyō-reki* (貞享暦) in the next year, and was officially used in Japan from AD 1685. It was the first theory of calendrical system produced in Japan. It mainly besed on the *Shoushi-li*, but it considered the time difference between China and Japan due to the terrestrial longitudinal difference between these two places, and also the change of the position of the apogee or perigee of the sun since the time of the *Shoushi-li*. In AD 1684, a new post "*tenmon-kata*" (天文方, [shogunate] astronomer) was created, and *Shibukawa Harumi* was appointed to be the first "*tenmon-kata*".

Shibukawa recognized that the earth is round through a Chinese popular astronomical book entitled *Tianjing-huowen* (天經或問, *Tenkei-wakumon* in Japanese) of *You Yi* (游藝, or *You Ziliu* (游子六)), which was imported to Japan sometime around AD 1670's, and some other Western information brought by Jesuits to East Asia. Although books contain Western culture (particularly Christianity) were basically prohibited to import at that time in Japan, the *Tianjing-huowen* was fortunately allowed to import. So, *Shibukawa* could recognize that the earth is round, and estimate the time difference between China and Japan using the comparison of the predicted time of eclipses by Chinese calendars and the actually observed time in Japan. This is a practional contribution of *Shibukawa* which Seki could not do. That the earth is round had been informed to some Japanese people by Jesuits in the late 16th century, but it was not so widely known. *Shibukawa*'s *Jōkyō-reki* is the first Japanese official document in which the round earth theory is explicitly assumed.

Shibukawa also obtained the knowledge of the change of the position of the apogee or perigee of the sun through an information of Western astronomy mentioned in the *Tianjing-huowen*. So, *Shibukawa* incorporated this knowledge in his calendar. This is another practional contribution of Shibukawa which *Seki* could not do.

The *Tianjing-huowen* tells that the longitude of the solar perigee is 6° from the point of winter solstice. *Shibukawa* used "6 *do* (度) 44 *fun* (分) and a half" as the longitude of the solar perigee from the point of winter solstice. In the East Asian classical astronomy, one *do* is the arc on the celestial sphere which the mean sun moves in a day (that is about 1/365.25 of a circle), and, in the *Jōkyō-reki*, one *fun* is 1/100 *do*. So, *Shibukawa*'s value is about 6°.4. As the correct longitude of the solar perigee at that time according to the modern astronomy was about 7°.5 from the point of winter solstice, Shibukawa's value was fairly good.

Although *Shibukawa Harumi* could not understand the mathematical rationale of the *Shoushi-li* perfectly, his adoption of the western knowledge of the change of the longitude of the solar perigee made his *Jōkyō-reki* more accurate than the *Shoushi-li*.

Actually, at that time, China had already accepted much information of Western geocentric astronomy, and had used more accurate *Shixian-li* (時憲暦) since AD 1645. However, it was not possible for Japanese astronomers at that time to study this Chinese new calendar, because books on Western culture were prohibited to import by the shogunate government at that time.

Anyway, *Shibukawa*'s practical ability should not be underestimated.

5. Conclusion

Seki Takakazu and *Shibukawa Harumi*, who were contemporaneous, made great contributions to the development of Japanese mathematics and/or astronomy. *Seki* was a theoretician, and his ability of mathematics could not be rivalled. *Shibukawa* was a practician, and only his new devices, including the adoption of the new data of the longitude of the solar perigee, could lead his new calendar to success.

Seki and *Shibukawa* made firm foundation of the later dialectical development of the theoretical mathematics and the practical astronomy. Both of these branches are indispensable. Several different elements are contributing to the development of science.

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